# ELECTRICALLY CHARGED BLACK-HOLES FOR THE HETEROTIC STRING

# COMPACTIFIED ON A $(10-D)$-TORUS

Pablo M. Llatas[1]

*Department of Physics*
*University of California*
*Santa Barbara, CA 93106-9530*

We show that the most general stationary electrically charged black-hole solutions of the heterotic string compactified on a $(10-D)$-torus (where $D > 3$) can be obtained by using the solution generating transformations of Sen acting on the Myers and Perry metric. The conserved charges labeling these black-hole solutions are the mass, the angular momentum in all allowed commuting planes, and $36-2D$ electric charges. General properties of these black-holes are also studied.

May, 1996

[1] llatas@denali.physics.ucsb.edu

## 1. Introduction

Black-hole solutions emerging in string theory have been extensively studied in the recent literature. One of the reasons for such attention is the suggestion that the elementary massive string states (with Planck masses) could be identified with black-holes ([1] and references therein). This identification is not necessarily made in the same string theory but usually involves dual pictures: a classical black-hole solution of a string theory ("soliton") is identified with a quantum (bound or elementary) state on a dual theory. Identifications of this type have been recently employed to provide a statistical derivation of the Bekenstein-Hawking entropy by identifying the black-holes solutions with bound configurations of D-branes ([2,3,4,5,6,7]).

In the present work, we generate the most general rotating electrically charged black-hole solution of the heterotic string compactified on a $(10-D)$-torus (conforming the "no hair" theorems). We follow the work of [8], where the particular case $D=4$ was studied. Related works (using the same rotating technique to generate new solutions from a given one) can be found in [9,10,11,12], where cases of different dimensions, charges and number of rotation planes where studied. In the present work, we generate a family of solutions depending on a mass, $\left[\frac{D-1}{2}\right]$ angular momenta ($\left[\ \right]$ denotes the integer part) and $36-2D$ electrical charges. 16 electrical charges come from the $U(1)^{16}$ of the heterotic string in ten dimensions (on a general point on the moduli space of compactifications) and the remaining $2\cdot(10-D)$ electric charges come from the compactified dimensions (any time we compactify a spatial dimension there appears generically two new $U(1)$ gauge fields: the Kaluza-Klein $U(1)$ field comming from the metric $G_{\mu\nu}$ and the "winding" $U(1)$ field originating from the antisymmetric tensor field $B_{\mu\nu}$). These $1+\left[\frac{D-1}{2}\right]+36-2D$ are the largest number of parameters labeling these stationary black-holes conforming the "no hair" theorems. In section 2 we introduce the action for the heterotic string compactified on a $(10-D)$-torus. The solutions we are about to introduce satisfy the equations of motion derived from this action. In section 3 we describe the generalization to any $D$ of the solution generating technique of [8] which generates new solutions to the equations of motion from a given one. We will show that the generating boosts depend on $36-2D$ free parameters, in such a way that if we rotate the Myers and Perry metric [13] (which already depends on $1+\left[\frac{D-1}{2}\right]$ parameters corresponding to the mass and angular momenta associated to rotations in all commuting planes) we generate the most general solution we want to construct. In section 4 we carry out the rotation and obtain expressions for the family of new solutions. Finally, in section 5 we study general features of these general black-hole solutions, like the mass, electric charges, angular momentums, ergosphere and horizons.

## 2. The Heterotic String Compactified on a $(10-D)$-Torus.

The low energy effective theory of the ten-dimensional heterotic string corresponds to $N=1$ supergravity coupled to $U(1)^{16}$ super Yang-Mills (on a generic point of the space of compactification). The bosonic content of this theory is given by the metric $G_{AB}$, the dilaton $\Phi$, the antisymmetric tensor $B_{AB}$ and the $U(1)$ gauge bosons $A^I_A$ (where $A,B$ :

$0, 1, ..., 9$, and $I : 1, 2, .., 16$). When we compactify on a $(10-D)$-torus, the metric $G_{AB}$ induces the D-dimensional metric $G_{\mu\nu}$, $10-D$ $U(1)$ Kaluza-Klein gauge bosons $G_{\mu i}$ and $(11-D)(10-D)/2$ scalars $G_{ij}$ ( here $\mu, \nu : 0, 1, ..D-1$, and $i, j : 1, 2, ..10-D$). The gauge $U(1)$ bosons $A^I_A$ originate 16 $U(1)$ gauge bosons $A^I_\mu$ and $16\cdot(10-D)$ scalars $A^I_i$. Finally, the antisymmetric tensor field $B_{AB}$ induces the two form $B_{\mu\nu}$, $10-D$ $U(1)$ "winding" gauge bosons $B_{\mu i}$ and $(10-D)(9-D)/2$ scalars $B_{ij}$. Then, the total number of gauge bosons of the compactified theory is $36-2D$. These gauge bosons can be arranged in a column matrix of vectors $A^{(a)}_\mu$ (where $a : 1, .., 36-2D$). The total number of scalars (excluding the dilaton) is $260-36D+D^2$. These scalars can be arranged in a $(36-2D)\times(36-2D)$ matrix $M = M^T$ valued on the group G given by:

$$M \subset G, \qquad\qquad G = \frac{O(10-D, 26-D)}{O(26-D)\times O(10-D)} \tag{2.1}$$

This matrix fulfills $MLM^T = L$ where $L$ is the matrix given by:

$$L = \begin{pmatrix} -I_{26-D} & 0 \\ 0 & I_{10-D} \end{pmatrix} \tag{2.2}$$

($I_n$ is the $n\times n$ unit matrix). One can easily check that the dimension of $G$ is precisely $260-36D+D^2$, in such a way that the scalars of the theory (excluding the dilaton) fit in $G$. In terms of $\Phi$, $M$, $A^{(a)}_\mu$, $B_{\mu\nu}$ and $L$, the action for the heterotic string compactified on a $(10-D)$-torus takes the form [14]:

$$\begin{aligned} S = C\int d^Dx\sqrt{-G}e^{-\Phi}\big[&R_G + G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{8}G^{\mu\nu}Tr(\partial_\mu ML\partial_\nu ML) \\ &- \frac{1}{12}G^{\mu\mu'}G^{\nu\nu'}G^{\rho\rho'}H_{\mu\nu\rho}H_{\mu'\nu'\rho'} \\ &- G^{\mu\mu'}G^{\nu\nu'}F^{(a)}_{\mu\nu}(LML)_{ab}F^{(b)}_{\mu'\nu'}\big] \end{aligned} \tag{2.3}$$

where

$$F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu \tag{2.4}$$

and

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + 2A^{(a)}_\mu L_{ab}F^{(b)}_{\nu\rho} + \text{cyclic.} \tag{2.5}$$

It is straightforward to check that this action is invariant under global rotations $\Omega$ leaving the matrix $L$ invariant ($\Omega L\Omega^T = L$):

$$M \to \Omega M\Omega^T, \qquad A^{(a)}_\mu \to \Omega_{ab}A^{(b)}_\mu \tag{2.6}$$

Under these rotations, $G_{\mu\nu}$, $B_{\mu\nu}$ and $\Phi$ remain invariant.

## 3. The Solution-Generating Technique, "Bar" Fields.

Here we describe in some detail the solution-generating technique by following the work of Sen [8] where the case $D = 4$ was studied (see also [12,9,10,11]). This technique

can be used to, given one time-independent solution of the equations of motion of the action (2.3), generate a new family of time-independent solutions. If we restrict ourselves to time-independent backgrounds we find two new $U(1)$ gauge fields in the theory: $B_{0m}$ and $\frac{G_{0m}}{G_{00}}$. The first $U(1)$ gauge field is related to the time-independent gauge invariance $\delta B_{m0} = \partial_m \lambda_0 - \partial_0 \lambda_m = \partial_m \lambda_0$. The second gauge field is related to invariance under local time-independent translation of the time coordinate. One can add these two new $U(1)$ gauge fields to our $1 \times (36-2D)$ column matrix of vectors $A_\mu^{(a)}$ and define the $1 \times (38-2D)$ matrix of vectors $\bar{A}_\mu^{(\bar{a})}$ ($\bar{a} : 1, 2, ..38-2D$) given by:

$$
\begin{aligned}
\bar{A}_n^{(a)} &\equiv A_n^{(a)} - \frac{G_{0n}}{G_{00}} A_0^{(a)} \qquad 1 \leq a \leq 36-2D \\
\bar{A}_n^{(37-2D)} &\equiv \frac{1}{2} \frac{G_{0n}}{G_{00}} \\
\bar{A}_n^{(38-2D)} &\equiv \frac{1}{2} B_{0n} + A_0^{(a)} L_{ab} \bar{A}_n^{(b)}
\end{aligned}
\tag{3.1}
$$

(note that $A_0^{(a)}$ is a scalar under time-independent general coordinate transformations).

Also, the $(36-2D) \times (36-2D)$ $G$-valued matrix $M$ is promoted to a $(38-2D) \times (38-2D)$ matrix $\bar{M}$. The elements of the $\bar{M}$ matrix are given by ($\bar{M} = \bar{M}^T$):

$$
\begin{aligned}
\bar{M}_{ab} &\equiv M_{ab} + 4 \frac{A_0^{(a)} A_0^{(b)}}{G_{00}} \qquad 1 \leq a, b \leq 36-2D \\
\bar{M}_{a,37-2D} &\equiv -2 \frac{A_0^{(a)}}{G_{00}} \qquad 1 \leq a \leq 36-2D \\
\bar{M}_{37-2D,37-2D} &\equiv \frac{1}{G_{00}} \\
\bar{M}_{a,38-2D} &\equiv 2(ML)_{ab} A_0^{(b)} + 4 \frac{(A_0^{(b)} L_{bc} A_0^{(c)})}{G_{00}} A_0^{(a)} \qquad 1 \leq a \leq 36-2D \\
\bar{M}_{37-2D,38-2D} &\equiv -2 \frac{(A_0^{(b)} L_{bc} A_0^{(c)})}{G_{00}} \\
\bar{M}_{38-2D,38-2D} &\equiv G_{00} + 4 A_0^{(b)} (LML)_{bc} A_0^{(c)} + 4 \frac{(A_0^{(b)} L_{bc} A_0^{(c)})^2}{G_{00}}
\end{aligned}
\tag{3.2}
$$

Also, one defines a "bar" metric by:

$$
\bar{G}_{nm} \equiv G_{nm} - \frac{G_{0n} G_{0m}}{G_{00}} \tag{3.3}
$$

and a "bar" antisymmetric tensor:

$$
\bar{B}_{nm} \equiv \frac{1}{2} B_{nm} + \frac{G_{0n}}{G_{00}} (A_m^{(a)} L_{ab} A_0^{(b)} - \frac{1}{2} B_{0m}) - (n \leftrightarrow m) \tag{3.4}
$$

Finally, the "bar" dilaton is defined through:

$$\bar{\Phi} = \Phi - \frac{1}{2}\ln(-G_{00}) \tag{3.5}$$

For time-independent field configurations, the action (2.3) can be written in terms of the "bar" fields as:

$$\begin{aligned} S = C\int dt \int d^{D-1}x\sqrt{-\bar{G}}e^{-\bar{\Phi}}\big[R_{\bar{G}} + \bar{G}^{mn}\partial_m\bar{\Phi}\partial_n\bar{\Phi} + \frac{1}{8}\bar{G}^{mn}Tr(\partial_m\bar{M}L\partial_n\bar{M}L) \\ - \frac{1}{12}\bar{G}^{mm'}\bar{G}^{nn'}\bar{G}^{ll'}\bar{H}_{mnl}\bar{H}_{m'n'l'} \\ - \bar{G}^{mm'}\bar{G}^{nn'}\bar{F}^{(\bar{a})}_{mn}(\bar{L}\bar{M}\bar{L})_{\bar{a}\bar{b}}\bar{F}^{(\bar{b})}_{m'n'}\big] \end{aligned} \tag{3.6}$$

where now:

$$\bar{F}^{(\bar{a})}_{mn} = \partial_m\bar{A}^{(\bar{a})}_n - \partial_n\bar{A}^{(\bar{a})}_m \tag{3.7}$$

$$\bar{H}_{mnl} = \partial_m\bar{B}_{nl} + 2\bar{A}^{(\bar{a})}_m\bar{L}_{\bar{a}\bar{b}}\bar{F}^{(\bar{b})}_{nl} + \text{cyclic.} \tag{3.8}$$

$(1 \le \bar{a}, \bar{b} \le 38 - 2D)$ and $\bar{L}$ is given by:

$$\bar{L} = \begin{pmatrix} L & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{3.9}$$

($L$ is the one in (2.2)). Again, one straightforwardly verifies that this action is invariant under the $O(11-D, 27-D)$ rotations leaving the matrix $\bar{L}$ invariant ($\bar{\Omega}\bar{L}\bar{\Omega}^T = \bar{L}$):

$$\begin{aligned} \bar{M} &\rightarrow \bar{M}' = \bar{\Omega}\bar{M}\bar{\Omega}^T \\ \bar{A}^{(\bar{a})}_n &\rightarrow \bar{A}'^{(\bar{a})}_n = \bar{\Omega}_{\bar{a}\bar{b}}\bar{A}^{(\bar{b})}_n \end{aligned} \tag{3.10}$$

Once more, $\bar{\Phi}$, $\bar{G}_{nm}$ and $\bar{B}_{nm}$ remain invariant under this $O(11-D, 27-D)$ rotation.

The spirit of the solution generating technique is the following. The rotation (3.10) in the space of time-independent backgrounds mixes the "genuine" initial $(36-2D)$ $U(1)$ gauge fields $A^{(a)}_\mu$ with the components $G_{00}$ and $G_{0n}$ of the metric tensor and the components $B_{0n}$ of the antisymmetric tensor (as can be seen in (3.1)). Then, through this mixing one is able to generate new non-trivial solutions to the equations of motion of (2.3) (non-trivial means that we do not produce equivalent solutions). Note that due to the fact that we do not have magnetic charges (monopoles) the gauge fields are well defined globally and so, the new metrics obtained after rotation (with whom the original gauge fields mix) are also well defined.

Let us now prove that the non-trivial generating rotations leaving invariant the asymptotic behaviour of the solution are labeled by $36-2D$ free parameters. As we noted above, not all the $O(11-D, 27-D)$ rotations are allowed. $\bar{\Omega}$ must satisfy $\bar{\Omega}\bar{L}\bar{\Omega}^T = \bar{L}$. Let us study this condition (we follow the lines of [8]). The first observation is that the $U$ matrix given by:

$$
U=\begin{pmatrix} I_{36-2D} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{3.11}
$$

diagonalizes $\bar{L}$:

$$
\bar{L}_d \equiv U\bar{L}U^T = \begin{pmatrix} -I_{26-D} & 0 & 0 & 0 \\ 0 & I_{10-D} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{3.12}
$$

Then, we can write the condition $\bar{\Omega}\bar{L}\bar{\Omega}^T = \bar{L}$ as ($U^TU = I_{38-2D}$):

$$
\bar{\Omega}_d \bar{L}_d \bar{\Omega}_d^T = \bar{L}_d \tag{3.13}
$$

where we have defined $\bar{\Omega}_d \equiv U\bar{\Omega}U^T$. In an asymptotically free space-time (where, at infinity, $G_{\mu\nu} \to \eta_{\mu\nu}$ and $A_\mu^{(a)} \to 0$), $\bar{M}$ has the asymptotic form:

$$
\bar{M} \to \begin{pmatrix} I_{36-2D} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3.14}
$$

(where we take $M = I_{36-D}$). From (3.12) and (3.14) we conclude [8] that $\bar{\Omega}_d$ (and then, $\bar{\Omega}$ itself) belongs to a subgroup $O(1, 26-D) \times O(10-D, 1)$ of $O(11-D, 27-D)$. The $O(1, 26-D)$ rotates the first $26-D$ elements and the element $37-2D$ of a column vector between themselves whereas the $O(10-D, 1)$ rotates the elements $27-D, .., 36-2D$ and $38-2D$ of a column vector between themselves. However, not all the elements of the subgroup $O(1, 26-D) \times O(10-D, 1)$ generate new solutions. We have to quotient by the global symmetry transformations $\Omega$ in (2.6) of the action (2.3) that do not generate new solutions. Then, we have to quotient $O(1, 26-D) \times O(10-D, 1)$ by the $O(26-D)$ rotations mixing the first $26-D$ elements of a column vector between themselves and the $O(10-D)$ rotations mixing the elements $27-D, .., 36-2D$ between themselves. Summarizing, the rotation group in the $38-2D$ internal space generating inequivalent solutions of the equations of motion associated to the action (2.3) is given by $\bar{G}$:

$$
\bar{G} = \frac{O(1, 26-D) \times O(10-D, 1)}{O(26-D) \times O(10-D)} \tag{3.15}
$$

One can easily verify that the dimension of $\bar{G}$ is precisely $36-2D$.

The strategy to generate the most general electrically charged black-hole solutions in the heterotic string compactified on a $(10-D)$-torus is now clear. First, we take as the generating solution the one given by:

$$
\begin{aligned}
G_{\mu\nu} &= G_{\mu\nu}^{MP} \\
M &= I_{36-2D} \\
\Phi = B_{\mu\nu} &= A_\mu^{(a)} = 0
\end{aligned} \tag{3.16}
$$

where $G^{MP}_{\mu\nu}$ is the metric of Myers and Perry (see next section), which from now on we will refer as the MP metric. This metric is designed for dimensions $D > 3$ and we will assume from the rest of the paper that this is the case. The MP metric already depends on $1 + \left[\frac{D-1}{2}\right]$ parameters: the mass and the angular momentums associated to the maximun allowed number of rotating planes. $M$ has to be an element of $G$ and $I_{36-2D}$ is the simplest choice. (3.16) is trivially a solution of the equations of motion of the action (2.3) (essentially, the constant fields satisfy automatically the equations of motion leaving as the only non-trivial equations the Einstein equations in vacuum, to which $G^{MP}_{\mu\nu}$ is a solution).

The next step is to use the solution (3.16) to define the "bar fields" (3.1) and (3.2). In particular $\bar{A}^{(\bar{a})}_n$ is given by (substitute (3.16) in the definitions (3.1) and (3.2)):

$$\bar{A}^{(\bar{a})}_n = \frac{1}{2}\frac{G_{0n}}{G_{00}}\delta_{\bar{a},37-2D} \tag{3.17}$$

and $\bar{M}$ is given by:

$$\bar{M} \rightarrow \begin{pmatrix} I_{36-2D} & 0 & 0 \\ 0 & (G_{00})^{-1} & 0 \\ 0 & 0 & G_{00} \end{pmatrix} \tag{3.18}$$

One then performs the $\bar{\Omega}$ rotation (depending on $36-2D$ free parameters) and gets $\bar{A}'^{(\bar{a})}_n$ and $\bar{M}'_{\bar{a}\bar{b}}$. Finally one goes backwards in the relations (3.1) and (3.2) and recovers the new solutions $G'_{\mu\nu}$, $M'$, $\Phi'$, $A'^{(a)}_\mu$ and $B'_{\mu\nu}$. The metric $G'_{\mu\nu}$ so obtained depends on $1 + \left[\frac{D-1}{2}\right] + 36 - 2D$ parameters, and then is the most general electrically charged black-hole solution allowed conforming the "no hair" theorems.

In the next section we explicitly carry out the $\bar{G}$ rotation and find the new solutions.

## 4. The General Solutions.

Let us first suppose that we have performed the rotations $\bar{\Omega}$ in (3.10) and obtained the vector $\bar{A}'^{(\bar{a})}_n$ and the matrix $\bar{M}'_{\bar{a}\bar{b}}$. We can use the relations defining the "bar fields" in (3.1) and (3.2) to obtain the new solutions in terms of these $\bar{A}'^{(\bar{a})}_n$ and $\bar{M}'_{\bar{a}\bar{b}}$. For instance, in (3.2) we see that $G'_{00}$ is just given by:

$$G'_{00} = \frac{1}{\bar{M}'_{37-2D,37-2D}} \tag{4.1}$$

As another example, we easily obtain $A'^{(a)}_0$ from (3.2):

$$A'^{(a)}_0 = -\frac{G'_{00}}{2}\bar{M}'_{a,37-2D} = -\frac{\bar{M}'_{a,37-2D}}{2\bar{M}'_{37-2D,37-2D}} \qquad 1 \leq a \leq 36-2D \tag{4.2}$$

In a similar way one can compute the expressions for all the fields of the new solutions in terms of the elements of the rotated vector $\bar{A}'^{(\bar{a})}_n$ and the rotated matrix $\bar{M}'_{\bar{a}\bar{b}}$. Let us collect the result. The new metric is given by:

$$G'_{00} = \frac{1}{\bar{M}'_{37-2D,37-2D}}, \qquad G'_{0n} = 2\frac{\bar{A}'^{(37-2D)}_n}{\bar{M}'_{37-2D,37-2D}}$$
$$G'_{nm} = G_{nm} - \frac{G_{0n}G_{0m}}{G_{00}} + 4\frac{\bar{A}'^{(37-2D)}_n \bar{A}'^{(37-2D)}_m}{\bar{M}'_{37-2D,37-2D}} \tag{4.3}$$

The dilaton is given by:

$$\Phi' = \Phi - \frac{1}{2}\ln(G_{00}\bar{M}'_{37-2D,37-2D}) \tag{4.4}$$

The expressions for the $U(1)$ gauge fields are:

$$A'^{(a)}_0 = -\frac{1}{2}\frac{\bar{M}'_{a,37-2D}}{\bar{M}'_{37-2D,37-2D}} \qquad 1 \leq a \leq 36-2D$$
$$A'^{(a)}_n = \bar{A}'^{(a)}_n - \frac{\bar{M}'_{a,37-2D}}{\bar{M}'_{37-2D,37-2D}}\bar{A}'^{(37-2D)}_n \tag{4.5}$$

The scalar fields of the new solution are given by:

$$M'_{ab} = \bar{M}'_{ab} - \frac{\bar{M}'_{a,37-2D}\bar{M}'_{b,37-2D}}{\bar{M}'_{37-2D,37-2D}} \qquad 1 \leq a,b \leq 36-2D \tag{4.6}$$

And finally, the new antisymmetric tensor is given by:

$$B'_{0n} = 2\bar{A}'^{(38-2D)}_n + \frac{\bar{M}'_{a,37-2D}}{\bar{M}'_{37-2D,37-2D}}L_{ab}\bar{A}'^{(b)}_n$$
$$B'_{nm} = \frac{1}{2}B_{nm} + \frac{G_{0n}}{G_{00}}A^{(a)}_m L_{ab}A^{(b)}_0 + B_{0n}\frac{G_{0m}}{2G_{00}}$$
$$+ \frac{1}{\bar{M}'_{37-2D,37-2D}}\bar{A}'^{(37-2D)}_n \bar{A}'^{(a)}_m L_{ab}\bar{M}'_{b,37-2D} + 2\bar{A}'^{(37-2D)}_n \bar{A}'^{(38-2D)}_m - (n \leftrightarrow m) \tag{4.7}$$

(note that this last expression simplifies in our case, because we are going to rotate the solution (3.16) where $\Phi = A^{(a)}_\mu = B_{\mu\nu} = 0$). What remains now is to compute the rotated vector $\bar{A}'^{(\bar{a})}_n$ and matrix $\bar{M}'_{\bar{a}\bar{b}}$ and substitute the result in the expressions (4.3)-(4.6) to obtain the final solution. In (3.15) we saw that the generating boosts are elements of the group

$$\bar{\Omega} \in \frac{O(1,26-D)\times O(10-D,1)}{O(26-D)O(10-D)} \tag{4.8}$$

We also saw that $\bar{\Omega}_d = U\bar{\Omega}U^T$ is such that the $O(1,26-D)$ rotates the first $26-D$ elements and the element $37-2D$ between themselves whereas the $O(10-D,1)$ rotates the elements $27-D,..,36-2D$ and $38-2D$ between themselves. The $O(26-D)$ and $O(10-D)$ that we are quotienting out are the rotations between the first $26-D$ elements

and the rotations between the elements $27-D,..,36-2D$ respectively. The simplest way to parametrize this $\bar{\Omega}_d$ is the following [8]:

$$\bar{\Omega}_d = U\Omega U^T = R(\bar{p},\bar{q})\cdot B(\alpha,\beta) \tag{4.9}$$

where $B(\alpha,\beta)$ performs a boost of angle $\alpha$ between the elements $26-D$ and $37-2D$ and a boost of angle $\beta$ between the elements $36-2D$ and $38-2D$:

$$\begin{aligned} B(\alpha,\beta)_{\bar{a},\bar{b}} = \delta_{\bar{a},\bar{b}} &+ (\cosh\alpha-1)(\delta_{\bar{a},26-D}\delta_{\bar{b},26-D}+\delta_{\bar{a},37-2D}\delta_{\bar{b},37-2D}) \\ &+ \sinh\alpha(\delta_{\bar{a},26-D}\delta_{\bar{b},37-2D}+\delta_{\bar{a},37-2D}\delta_{\bar{b},26-D}) \\ &+ (\cosh\beta-1)(\delta_{\bar{a},36-2D}\delta_{\bar{b},36-2D}+\delta_{\bar{a},38-2D}\delta_{\bar{b},38-2D}) \\ &+ \sinh\beta(\delta_{\bar{a},36-2D}\delta_{\bar{b},38-2D}+\delta_{\bar{a},38-2D}\delta_{\bar{b},36-2D}) \end{aligned} \tag{4.10}$$

and $R(\bar{p},\bar{q})$ is given by $R(\bar{p},\bar{q}) = R_{26-D}(\bar{p})\oplus R_{10-D}(\bar{q})\oplus I_2$ where $R_N(\bar{k})$ is a $N$-dimensional rotation matrix rotating the $N$-dimensional column unit vector $(0,0,..,0,1)$ to the $N$-dimensional unit vector $(k_1,k_2,..,k_N)$:

$$\begin{aligned} [R_N(\bar{k})]_{ij} =& \delta_{ij}\big[\big(1-\frac{{k_i}^2}{1+k_N}\big)(1-\delta_{iN})+k_N\delta_{iN}\big] \\ &+(1-\delta_{ij})\big[(\delta_{iN}+\delta_{jN}-1)\frac{k_ik_j}{1+k_N}+k_i\delta_{jN}-k_j\delta_{iN}\big] \end{aligned} \tag{4.11}$$

As a consistency check of this parametrization of the generating boosts, note that the total number of parameters of the matrix $\bar{\Omega}_d$ are $25-D$ coming from the unit vector $\bar{p}$, $9-D$ coming from the unit vector $\bar{q}$, and the two boost angles $\alpha$ and $\beta$, making a total of $36-2D$ parameters (precisely the dimension of the group (4.8)).

Now, we are in position to perform the boost of the vector $\bar{A}_n^{(\bar{a})}$ and the matrix $\bar{M}_{\bar{a}\bar{b}}$. First, we boost $\bar{A}_n^{\prime(\bar{a})}$. We need to compute:

$$\bar{A}_\mu^{\prime(\bar{a})} = \bar{\Omega}_{\bar{a}\bar{b}}\bar{A}_\mu^{(\bar{b})} = (U^TR(\bar{p},\bar{q})B(\alpha,\beta)U)_{\bar{a}\bar{b}}\bar{A}_\mu^{(\bar{b})} \tag{4.12}$$

From our expressions (3.11), (4.10), (4.11), and (3.17) one gets the following result:

$$\begin{aligned} \bar{A}_n^{\prime(a)} &= \frac{\sinh\alpha}{2\sqrt{2}}\frac{G_{0n}}{G_{00}}p^a \qquad 1\le a\le 26-D \\ \bar{A}_n^{\prime(a+26-D)} &= \frac{\sinh\beta}{2\sqrt{2}}\frac{G_{0n}}{G_{00}}q^a \qquad 1\le a\le 10-D \\ \bar{A}_n^{\prime(37-2D)} &= \frac{1}{4}(\cosh\alpha+\cosh\beta)\frac{G_{0n}}{G_{00}} \\ \bar{A}_n^{\prime(38-2D)} &= \frac{1}{4}(\cosh\alpha-\cosh\beta)\frac{G_{0n}}{G_{00}} \end{aligned} \tag{4.13}$$

Now, we compute $\bar{M}'$. We need to calculate, in this case:

$$\bar{M}' = \bar{\Omega}\bar{M}\bar{\Omega}^T = U^TR(\bar{p},\bar{q})B(\alpha,\beta)U\bar{M}U^TB^T(\alpha,\beta)R^T(\bar{p},\bar{q})U \tag{4.14}$$

The result is:

$$
\begin{aligned}
\bar{M}'_{ab} &= \delta_{ab} + (1+G^+)\sinh^2\alpha p^a p^b \qquad 1 \leq a,b \leq 26-D \\
\bar{M}'_{a,b+26-D} &= \sinh\alpha \sinh\beta G^- p^a q^b \qquad 1 \leq a \leq 26-D,\ 1 \leq b \leq 10-D \\
\bar{M}'_{a+26-D,b+26-D} &= \delta_{ab} + (1+G^+)\sinh^2\beta q^a q^b \qquad 1 \leq a,b \leq 10-D \\
\bar{M}'_{a,37-2D} &= \frac{a_1^+}{G_{00}} p^a \qquad 1 \leq a \leq 26-D \\
\bar{M}'_{a+26-D,37-2D} &= \frac{a_2^+}{G_{00}} q^a \qquad 1 \leq a \leq 10-D \\
\bar{M}'_{37-2D,37-2D} &= \frac{\Delta}{G_{00}} \\
\bar{M}'_{a,38-2D} &= \frac{a_1^-}{G_{00}} p^a \qquad 1 \leq a \leq 26-D \\
\bar{M}'_{a+26-D,38-2D} &= \frac{a_2^-}{G_{00}} q^a \qquad 1 \leq a \leq 10-D \\
\bar{M}'_{37-2D,38-2D} &= \bar{M}'_{38-2D,38-2D} = \frac{\delta}{G_{00}}
\end{aligned}
\tag{4.15}
$$

where we have defined:

$$
\begin{aligned}
G^\pm &\equiv \frac{1}{2}(\frac{1}{G_{00}} \pm G_{00}) \\
a_1^\pm &\equiv \frac{\sinh\alpha}{2\sqrt{2}} \left[\cosh\alpha \pm \cosh\beta + 2\cosh\alpha G_{00} + (\cosh\alpha \mp \cosh\beta){G_{00}}^2\right] \\
a_2^\pm &\equiv \frac{\sinh\beta}{2\sqrt{2}} \left[\cosh\alpha \pm \cosh\beta \pm 2\cosh\beta G_{00} - (\cosh\alpha \mp \cosh\beta){G_{00}}^2\right] \\
\Delta &\equiv \frac{1}{4}(\cosh\alpha + \cosh\beta)^2 + \frac{G_{00}}{2}(\sinh^2\alpha + \sinh^2\beta) + \frac{G_{00}^2}{4}(\cosh\alpha - \cosh\beta)^2 \\
\delta &\equiv \frac{1}{4}(\cosh^2\alpha - \cosh^2\beta)(1+{G_{00}}^2) + \frac{1}{2}(\sinh^2\alpha - \sinh^2\beta)G_{00}
\end{aligned}
\tag{4.16}
$$

Now, we are ready to substitute these results in (4.1) -(4.7) to obtain the final solution. Here we collect the result. The boosted metric reads:

$$
\begin{aligned}
{G'}_{00} &= \frac{G_{00}}{\Delta}, \qquad {G'}_{0n} = \frac{1}{2}(\cosh\alpha + \cosh\beta)\frac{G_{0n}}{\Delta} \\
{G'}_{nm} &= G_{nm} + \left[\frac{(\cosh\alpha + \cosh\beta)^2}{4\Delta} - 1\right]\frac{G_{0n}G_{0m}}{G_{00}}
\end{aligned}
\tag{4.17}
$$

The boosted dilaton is:

$$
\Phi' = -\frac{1}{2}\ln\Delta \tag{4.18}
$$

The $U(1)$ gauge fields are given by:

$$
\begin{aligned}
A'^{(a)}_{0} &= -\frac{a_1^+}{2\Delta}p^a \qquad 1 \le a \le 26-D \\
A'^{(a+26-D)}_{0} &= -\frac{a_2^+}{2\Delta}q^a \qquad 1 \le a \le 10-D \\
A'^{(a)}_{n} &= \frac{1}{2}\Big[\frac{\sinh\alpha}{\sqrt{2}} - \frac{a_1^+}{2\Delta}(\cosh\alpha+\cosh\beta)\Big]\frac{G_{0n}}{G_{00}}p^a \\
A'^{(a+26-D)}_{n} &= \frac{1}{2}\Big[\frac{\sinh\beta}{\sqrt{2}} - \frac{a_2^+}{2\Delta}(\cosh\alpha+\cosh\beta)\Big]\frac{G_{0n}}{G_{00}}q^a
\end{aligned}
\tag{4.19}
$$

The boosted scalars are:

$$
\begin{aligned}
M'_{ab} &= \delta_{ab} + \Big[(1+G^+)\sinh^2\alpha - \frac{(a_1^+)^2}{\Delta G_{00}}\Big]p^a p^b \quad 1\le a,b \le 26-D \\
M'_{a+26-D,b+26-D} &= \delta_{ab} + \Big[(1+G^+)\sinh^2\beta - \frac{(a_2^+)^2}{\Delta G_{00}}\Big]q^a q^b \quad 1\le a,b \le 10-D \\
M'_{a,b+26-D} &= \Big[G^-\sinh\alpha\sinh\beta - \frac{a_1^+ a_2^+}{\Delta G_{00}}\Big]p^a q^b \quad 1\le a \le 26-D, 1\le b\le 10-D
\end{aligned}
\tag{4.20}
$$

Finally, the boosted antisymmetric tensor reads:

$$
\begin{aligned}
B'_{0n} &= \frac{1}{2}\Big(\cosh\alpha - \cosh\beta + \frac{1}{\sqrt{2}}\frac{a_2^+\sinh\beta - a_1^+\sinh\alpha}{\Delta}\Big)\frac{G_{0n}}{G_{00}} \\
B'_{nm} &= 0
\end{aligned}
\tag{4.21}
$$

At first sight, some of the previous expressions seems to be ill defined over the ergosphere (i.e., the surface defined by $G_{00}=0$). However, a closer inspection shows that all the fields are well defined when $G_{00}=0$ (see section 5).

Note that the previous expressions are valid for any metric $G_{\mu\nu}$ satisfying the Einstein equations in the vacuum. The only information that we have used so far is that (see (3.16)) $M = I_{36-2D}$ and $\Phi = B_{\mu\nu} = A^{(a)}_{\mu} = 0$. Now, to have the most general electrically charged black-hole solution of the heterotic string compactified on a $(10-D)$-torus we have to substitute the Myers and Perry metric $G^{MP}_{\mu\nu}$ in the relations (4.17)-(4.21). Here we give the MP metric in polar coordinates which are the best adapted to the symmetry of a rotating black-hole in $\left[\frac{D-1}{2}\right]$ commuting planes (with more than one plane of rotation, the spherical coordinates lead to complicated, long and asymmetrical expressions). On each plane of rotation (there are $\left[\frac{D-1}{2}\right]$) we select polar coordinates $(r_i,\theta_i)$ $(i : 1,..\left[\frac{D-1}{2}\right])$. When the spatial dimension $(D-1)$ is odd, we have to introduce another coordinate $z$, labeling the direction in which there is not rotation. $a_i$ is the rotation parameter along the plane labeled by the coordinates $(r_i,\theta_i)$. The MP metric is given by (remember that

$D > 3$) [13]:

$$
\begin{aligned}
&G^{MP}_{00} = h - 1, \qquad\qquad G^{MP}_{0r_i} = h\frac{\rho r_i}{\rho^2 + {a_i}^2}\\
&G^{MP}_{0\theta_i} = h\frac{a_i {r_i}^2}{\rho^2 + {a_i}^2}, \qquad\qquad G^{MP}_{0z} = \frac{(1-(-1)^{D-1})}{2} h\frac{z}{\rho}\\
&G^{MP}_{r_i r_j} = \delta_{ij} + h\frac{\rho^2 r_i r_j}{(\rho^2 + {a_i}^2)(\rho^2 + {a_j}^2)}\\
&G^{MP}_{\theta_i r_j} = h\frac{\rho a_i {r_i}^2 r_j}{(\rho^2 + {a_i}^2)(\rho^2 + {a_j}^2)}, \qquad G^{MP}_{r_i z} = \frac{(1-(-1)^{D-1})}{2} h\frac{z r_i}{\rho^2 + {a_i}^2}\\
&G^{MP}_{\theta_i\theta_j} = {r_i}^2\delta_{ij} + h\frac{a_i a_j {r_i}^2 {r_j}^2}{(\rho^2 + {a_i}^2)(\rho^2 + {a_j}^2)}\\
&G^{MP}_{\theta_i z} = \frac{(1-(-1)^{D-1})}{2} h\frac{a_i {r_i}^2 z}{\rho(\rho^2 + {a_i}^2)}, \qquad G^{MP}_{zz} = \frac{(1-(-1)^{D-1})}{2}\Big(1 + h\frac{z^2}{\rho^2}\Big)
\end{aligned}
\tag{4.22}
$$

In the previous expressions, $\rho = \rho(r_i, a_i)$ ($i : 1, ..\left[\frac{D-1}{2}\right]$) is the function defined by the constraint:

$$\sum_i \Big(\frac{{r_i}^2}{\rho^2 + {a_i}^2}\Big) + \frac{(1-(-1)^{D-1})}{2}\frac{z^2}{\rho^2} = 1 \tag{4.23}$$

and $h$ is given by:

$$h = \frac{(1+(-1)^{D-1})}{2}\frac{\mu\rho^2}{\Pi F} + \frac{(1-(-1)^{D-1})}{2}\frac{\mu\rho}{\Pi F} \tag{4.24}$$

being:

$$\Pi = \prod_i (\rho^2 + {a_i}^2), \qquad\qquad F = 1 - \sum_i \frac{{a_i}^2 {r_i}^2}{(\rho^2 + {a_i}^2)^2} \tag{4.25}$$

An important consequence of the constraint (4.23) (useful when computing the conserved charges at infinity) is that:

$$\lim_{r_{i_1}, \cdot\cdot, r_{i_L} \to \infty} \rho^2 = \sum_{i \in U_L} r_i^2 \tag{4.26}$$

where $U_L = (r_{i_1}, \cdot\cdot, r_{i_L})$ is any subset of the radial polar coordinates $r_i$ ($i : 1, .., \left[\frac{D-1}{2}\right]$).

It is convenient to remark that the MP metric does not depend on the time $t$ nor the polar angles $\theta_i$; it is obviously invariant under time translations and $\theta_i$ translations. The Killing field associated to the first isometry is:

$$\xi^t = \frac{\partial}{\partial t} \tag{4.27}$$

and the associated conserved charge is the mass. On the other hand, the Killing field associated to translations on $\theta_i$ is:

$$\xi^i = \frac{\partial}{\partial \theta_i} \tag{4.28}$$

and the associated conserved charges are the angular momenta due to the rotations along the corresponding planes. These isometries are also isometries of the rotated metric (4.17), because it is still independent of the time $t$ and the polar angles $\theta_i$. They play an important role when computing the mass, the angular momentums and horizon of the general electrically charged black-hole.

## 5. Conserved Charges, Ergosphere and Horizons.

In this section we are going to discuss general properties of the electrically charged black-holes that we have obtained in (4.17)-(4.21). First, we compute the conserved charges of the solution, starting with the mass and the angular momentum. The appropriate metric to use is the Einstein metric, related to (4.17) (from now on we take $G_{\mu\nu} = G^{PM}_{\mu\nu}$ in (4.17)) by the following relation:

$$G'^{E}_{\mu\nu} = e^{\frac{2\Phi'}{2-D}} G'_{\mu\nu}. \tag{5.1}$$

The ADM mass is the charge corresponding to the Killing vector field (4.27) and is related to the invariance of the system under global time-translations. One obtains:

$$M = \frac{1}{2}(1 + (D-3)\cosh\alpha\cosh\beta)\mu\Omega_{D-2} \tag{5.2}$$

where $\Omega_{D-2}$ is the area of the unit $(D-2)$-sphere. The angular momentum $J_i$ is the charge corresponding to the Killing vector field (4.28) and is related to invariance of the solution under global translations of the polar angle $\theta_i$. The result is:

$$J_i = \frac{1}{2}(\cosh\alpha + \cosh\beta)\mu a_i \Omega_{D-2} \tag{5.3}$$

For the electric charges $Q^{(a)}$ of the black-hole one obtains:

$$\begin{aligned} Q^{(a)} &= \frac{\mu}{\sqrt{2}}(D-3)\sinh\alpha\cosh\beta p^a \Omega_{D-2}, \qquad 1 \le a \le 26 - D \\ Q^{(a+26-D)} &= \frac{\mu}{\sqrt{2}}(D-3)\sinh\beta\cosh\alpha q^a \Omega_{D-2}, \qquad 1 \le a \le 10 - D. \end{aligned} \tag{5.4}$$

Let us now localize the ergosphere. The ergosphere is the surface of space-time defined by the equation $G'^{E}_{00} = 0$. In our case, from (5.1) and (4.17) we get:

$$G'^{E}_{00} = \Delta^{\frac{3-D}{D-2}} G^{MP}_{00} = 0 \tag{5.5}$$

This equation, for finite boosts angles $\alpha$ and $\beta$, has the solution $G^{MP}_{00} = 0$. Then, the ergosphere of the boosted black-holes are at the same place as the ergosphere of the MP metric. From (4.22) we find that the ergosphere is in the region where $h = 1$, and then, is defined by the equation (see (4.24)):

$$\begin{aligned} \Pi F &= \mu\rho^2 \qquad \text{for D} - 1 \text{ even} \\ \Pi F &= \mu\rho \qquad \text{for D} - 1 \text{ odd} \end{aligned} \tag{5.6}$$

One can check that all the fields are well defined over the ergosphere and that, most notably, the scalar fields take constant values there (as one can easily show from (4.18) and (4.20)):

$$
\begin{aligned}
\Phi'|_{erg} &= -\ln\Big(\frac{\cosh\alpha + \cosh\beta}{2}\Big) \\
M'_{ab}|_{erg} &= \delta_{ab} + \sinh^2\alpha\, p^a p^b, \qquad 1 \le a, b \le 26 - 2D \\
M'_{a+26-D,b+26-D}|_{erg} &= \delta_{ab} + \sinh^2\beta\, q^a q^b, \qquad 1 \le a, b \le 10 - D \\
M'_{a,b+26-D}|_{erg} &= 0, \qquad 1 \le a \le 26 - d, \quad 1 \le b \le 10 - D.
\end{aligned}
\tag{5.7}
$$

Also, the temporal component of the gauge fields are constant on the ergosphere:

$$
\begin{aligned}
\phi^{(a)} \equiv A'^{(a)}_0|_{erg} &= -\frac{1}{\sqrt{2}}\frac{\sinh\alpha}{\cosh\alpha + \cosh\beta} p^a, \qquad 1 \le a \le 26 - D \\
\phi^{(a+26-D)} \equiv A^{(a+26-D)}_0|_{erg} &= -\frac{1}{\sqrt{2}}\frac{\sinh\beta}{\cosh\alpha + \cosh\beta} q^a, \qquad 1 \le a \le 10 - D
\end{aligned}
\tag{5.8}
$$

Surprisely, we will see below that $\phi^{(j)}$ $(1 \le j \le 36 - 2D)$ coincides with the electrostatic potential in the event horizon.

Let us now talk about the horizons of the electrically charged black-holes. We are going to compute the location of the horizons by using the covariant method of Carter [15]. First we construct the "Killing form" (a $\left[\frac{D-1}{2}\right] + 1$ form):

$$
K = K_{\mu_0\mu_1,..,\mu_p} dx^{\mu_0} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p} \tag{5.9}
$$

where we have defined $p \equiv \left[\frac{D-1}{2}\right]$ and:

$$
K_{\mu_0\mu_1,..,\mu_p} \equiv \xi^t_{[\mu_0} \xi^1_{\mu_1} \cdots \xi^p_{\mu_p]} \tag{5.10}
$$

($\xi^t$ and $\xi^i$ are the Killing fields given in (4.27) and (4.28)). The horizon is then located at the hypersurface in space-time where the norm of this form vanishes ($|K|^2 = 0$). Let us denote by $|K'|^2$ the norm of the Killing form computed with the metric (4.17) (where $G_{\mu\nu} = G^{MP}_{\mu\nu}$) and by $|K^{MP}|^2$ the norm computed with the MP metric. From (4.17) one finds:

$$
|K'|^2 = \frac{|K^{MP}|^2}{\Delta} \tag{5.11}
$$

As a result, for finite rotation angles $\alpha$ and $\beta$, the horizon of the electrically charged black-holes are localized in the same place as the horizon of the MP metric (however, the area of the horizon changes due to the change of the metric). The solution to the equation $|K^{MP}|^2 = 0$ is given by:

$$
\begin{aligned}
\Pi &= \mu\rho \qquad \text{for D even} \\
\Pi &= \mu\rho^2 \qquad \text{for D odd}
\end{aligned}
\tag{5.12}
$$

These equations has been studied by Myers and Perry ([13]). Looking at the form of $\Pi$ (4.25) we see that (5.12), for $D$ even, is a polynomial equation of degree $D-2$. This means that, if they gave solution(s), one can write an explicit formula for them in the cases $D = 4, 6$ *. For $D$ odd (5.12) is a polynomial of degree $\frac{D-1}{2}$ in $\rho^2$. Therefore, the solution(s) (if any) can also be written explicitly for $D = 5, 7, 9$. The cases $D = 8, 10$ have to be studied separately.

In [13] it is shown that (5.12) can have, at most, two solutions $\rho_H^{\pm}$ leading to the inner horizon $\rho_H^-$ and the event $\rho_H^+$ horizon. The degenerate case with only one solution ($\rho_H^+ \to \rho_H^-$) gives the extremal black-hole solution. Finally, the cases with no solution produce naked singularities.

We now wish to compute the physical quantities relevant in black-hole thermodynamics. First, we compute the null generator $\tilde{k}'$ of the event horizon of the charged black-holes (from now on primes denotes quantities associated to the boosted black-hole whereas unprimed denote those associated to the unboosted MP metric). $\tilde{k}'$ is the combination of Killing fields (4.27) and (4.28) that becomes null on the event horizon. Then, we define:

$$\tilde{k}' = \xi^t - \sum_i \Omega'_i \xi^i, \qquad 1 \leq i \leq \Big[\frac{D-1}{2}\Big]. \tag{5.13}$$

Demanding on the event horizon that $|\tilde{k}'|^2 = G'^E{}_{\mu\nu}\tilde{k}'^\mu \tilde{k}'^\nu = 0$ (where $G'^E{}_{\mu\nu}$ is the metric given in (5.1)) we get:

$$\Omega'_i = \Big(\frac{2}{\cosh\alpha + \cosh\beta}\Big)\frac{a_i}{{\rho_H^+}^2 + a_i^2} \tag{5.14}$$

To obtain the previous result we have used the fact that, on the event horizon, from (4.24) and (5.12) we have:

$$h|_{\rho_H^+} = \frac{1}{F} \tag{5.15}$$

The electrostatic potential on the surface of the horizon is given by:

$$\phi^{(a)} = A'^{(a)}_\mu \tilde{K}'^\mu|_{\rho_H^+}. \tag{5.16}$$

Notably, from (4.19), (5.13) and (5.14) (and using again (5.15))one finds that the electrostatic potential on the horizon (which is constant) is precisely the temporal component of the gauge fields on the ergosphere (as we anticipated in (5.8)). This seems to be a general rule common to the solutions constructed using the generating technique, and we have checked that the same coincidence happens for the Kaluza-Klein black holes of [16]. Next, we compute the surface gravity $\kappa'$ of the rotated black-holes. $\kappa'$ is defined through the equation:

$$\tilde{k}'^\mu \nabla'_\mu \tilde{k}'^\nu = \kappa' \tilde{k}'^\nu \tag{5.17}$$

---

* Here we recall briefly the Galois' theorem: the roots of a generic polynomial in $\rho$ of order $n$ are soluble in terms of radical expressions only for $n = 2, 3, 4$.

evaluated on the horizon ($\nabla'$ is the covariant derivative with respect to the metric (5.1)). After a long (but trivial) computation we get a simple answer:

$$\kappa' = \frac{2}{\cosh\alpha + \cosh\beta}\kappa \tag{5.18}$$

where $\kappa$ is the surface gravity of the rotating black-holes without electric charges:

$$\begin{aligned} \kappa &= \frac{\partial_\rho \Pi - 2\mu\rho}{2\mu\rho^2}|_{\rho_H^+}, \qquad \text{for odd D} \\ \kappa &= \frac{\partial_\rho \Pi - \mu}{2\mu\rho}|_{\rho_H^+}, \qquad \text{for even D} \end{aligned} \tag{5.19}$$

Of course, $\rho_H^+$ is a constant on the event horizon and, as a result, $\kappa$ is also a constant there.

Finally, the area of the horizon is given by:

$$A'_H = (\cosh\alpha + \cosh\beta)\frac{\mu}{4\kappa}\big(D - 3 - 2\sum_i \frac{a_i}{{\rho_H^+}^2 + a_i^2}\big)\Omega_{D-2}. \tag{5.20}$$

The most efficient way of computing the area is by noting that $\kappa A_H$ (as is the case for $\sum_i \Omega_i J_i$) is an invariant under the generating boosts (this is also true for the Kaluza-Klein black holes of [16]). Then, using the result of Myers and Perry for the area of the unboosted case we arrive at (5.20).

The case of $\left[\frac{D-2}{2}\right]$ rotating planes and two electric charges was discussed in [17] (which appeared after completion of this work).

**Acknowledgements**

We would like to thank R. Emparan, G.T. Horowitz and D. Marolf for valuable discussions. This work is partly supported by a Spanish Government grant and partially supported by DOE-91ER40618 and DGICYT (PB93-0344).

## References


[1] A. Sen, Mod. Phys. Lett. A10(95)2081, hep-th/9504147.

[2] A. Strominger and C. Vafa, hep-th/9601029.

[3] C.G. Callan and J.M. Maldacena, hep-th/9602043.

[4] G.T. Horowitz and A. Strominger, hep-th/9602051.

[5] J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa, hepth/9602065.

[6] J.M. Maldacena and A. Strominger, hep-th/9603060.

[7] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger and C. Vafa, hep-th/9603078.

[8] A. Sen, Nucl. Phys. B440(95)421, hep-th/9411187.

[9] G.T. Horowitz and A. Sen hep-th/9509108

[10] M. Cvetic and D. Youm, hep-th/9512127.

[11] M. Cvetic and D. Youm, hep-th/9603100.

[12] A. Peet, hep-th/9506200.

[13] R.C. Myers and M.J. Perry, Ann. Phys. 172(86)304.

[14] J. Maharana and J. Schwarz, Nucl. Phys. B390(93)3, hep-th/9207016.

[15] B. Carter, J. Math. Phys. 10(69)70.

[16] J.H. Horne and G.T. Horowitz, Phy. Rev. Lett. D 46(1340).

[17] M. Cvetic and D. Youm, hep-th/9605051.